Paper ST-156

Optimizing the Marketing Mix

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1. ABSTRACT

Practitioners in the media industry are frequently faced with the problem of diverting limited resources among several alternative channels. The paper looks at this problem of optimal allocation of limited resources to maximize an objective function within the context of media mix modeling. If the objective function is linear or convex these problems can be easily solved by applying standard methods of linear or geometric programming. However, in the media world, non linear relationships between sales and media variables and advertising carry over effects can lead to non convex objective functions that make matters more complicated.

In this paper, we develop an alternative solution using an algorithm (which we deploy with the help of a SAS macro) that can be used to obtain a “close to” global solution for the optimal media mix. The algorithm uses an iteration method to obtain the value of the objective function for all potential allocations. The allocation that maximizes the objective function can then be chosen as the most favorable media mix. Since the method works with the transformed variables, non linear transformations and complicated carryover effects can be incorporated within this structure. This allows for a more realistic estimation of media impacts.

2. INTRODUCTION

Most businesses spend millions of dollars on advertising every year. According to data released by TNS Media Intelligence, total advertising expenditures in 2006 were $149.6 billion, an increase of 4.1% since 2005. The 2007 advertising market slowed down somewhat due to pessimism about general economic conditions; in spite of that, advertising expenditures in 2007 amounted to $148.99 billion, up 0.2% compared to 2006. In recent years, National TV has accounted for the majority (about 30%) of the advertisement budget, with magazines close behind at about 20%.

Although advertising expenditures have increased in recent years, marketers are increasingly cautious about the dollars spent on advertising and demand more accountability from their communication partners about their marketing spending, making it imperative for media planners to invest their marketing dollars in the best possible manner to maximize returns in terms of incremental sales. Seeking to justify their marketing spending, marketing planners in turn have solicited the help of statisticians and econometricians to quantify the effects of marketing spending as well as to direct the spending to the most efficient channels. Various marketing models have therefore been developed to help calculate the effectiveness (dollar sales per GRP) as well as the return on investment on advertising expenditures (dollar Sales per unit of expenditure) and provide direction and accountability to marketing spending. These models help to determine how much a business should spend on advertising in order to increase sales and get a positive return on their investment. The analytical and statistical methods that are used to quantify the effects of media and marketing efforts on a product's performance is referred to as “Marketing Mix Modeling”.

In order to reach various different audiences, media planners often have to allocate their resources between various media including Television, print, magazine, radio, internet, out of home etc. They may also distribute their advertising budget between different products. For instance, in the entertainment industry, the advertisement budget for a movie may be spent on the Theatrical version of the movie as well as on the DVD version. Therefore, an equally important topic of interest to marketing mix modeling professionals is how best to allocate a fixed budget among various different media instruments(or products) in order to maximize the incremental returns from that investment. The optimal portfolio finds the relative levels of the two or more media vehicles (or products, channels etc) that maximize retail sales while keeping a fixed budget. For example we might be required to find the mix between :15 and :30 second television spots that maximize retail sales at a given level of spending. Another problem may be to find the mix between spending on Theatrical media and spending on DVD media that maximizes the Total Revenue (Box office sales + DVD sales). Alternatively, we may be required to obtain the optimal allocation of spending between the various media types such as TV, Magazine, Newspaper and Online.

3. SHORTCOMINGS OF EXISTING METHODS

The problem of finding an optimal mix has received a good deal of attention in the marketing literature. Doyle and Sanders (1990) proposed a multiproduct advertising budgeting model which allowed product categories to have own and cross effects. This model can easily be extended to address the problem of allocation of a fixed budget between different media vehicles. However, most of these studies have focused on demand curves that are either linear or linear additive and have used demand functions which can be easily solved using algebraic methods for the optimal solution. Few studies that have used non linear demand and cost functions still assume that the objective function is convex or can be at least transformed into a convex functional.

In practice however, the media industry uses various transformations some of which may be non-linear and difficult to manipulate algebraically. For instance, practitioners in the media world often assume that the media response function is S-shaped, i.e. has an initial convex and subsequently a concave section. The theoretical reason behind this shape is that a so-called *threshold effect* takes place, i.e. the phenomena that marketing efforts are not effective until they exceed a certain minimum level (Hanssens et al. 2001, p. 113). Most managers and practitioners agree that advertising threshold effects exist so that there are levels of advertising below which there is essentially no sales response (Corkindale and Newall, 1978 ; Ambler ,1996). This has led various researchers to recognize an S shaped advertising response curve. When two or more media variables have such non-linear S shaped response curves the problem of choosing the optimal media mix among various media may have a non-convex objective function

Apart from the shape of the response curve, the dynamic nature of marketing adds to the complexities of the models. The effects of marketing campaigns do not usually end when the campaign is over. At least some of the effect of a campaign can be perceived in future periods. In other words, sales today will be determined not only by today's advertising but also by advertising expenditures in previous periods. These are called lagged effects or carryover effects of advertising. There can be lead effects of advertising as well. For instance, consumers sometimes anticipate a marketing stimulus and adjust their behavior before the stimulus actually occurs. Various different kinds of lagged or lead effects for the media variables are considered by marketing mix modeling practitioners and the one that enters the model may be a complicated transformation of the original variable. It is not uncommon for instance, to first create a lagged geometric series from the original media variable and then work with logs of this geometric series. In these situations it is very hard to come up with an algebraic solution for the optimal mix. The problem then resorts to one of finding the optimal mix through an iterative process by using Proc IML or a trial and error process by using a tool such as the Excel solver. However, the final solutions often depends on the initial conditions submitted and with non linearities in the model, these methods might end up finding a local optima rather than the global one.

Furthermore, each optimization technique under Proc IML requires a continuous objective function and almost all optimization subroutines (except the NLPNMS subroutine) require continuous first order derivatives of the objective function. It may be useful to have an algorithm which can work even without these restrictions. Moreover, even if a global solution is obtained using the above methods, the solution may not make business sense. Looking at several alternative solutions which make more business sense may help to choose the best marketing mix.

3.1. AN ILLUSTRATIVE EXAMPLE

We illustrate the above issues with the help of the following example. The S shaped curves in figure 1 shows the impact of various media variables on Sales. The media variables have been labeled x, y and z.. For example, the line labeled Sales_x depicts the incremental sales that will result when impressions of the media variable called x range from 0 to 205 per week. In this case, x may be weekly TV GRPs, y may be magazine impressions per week and z may be Online impressions per week. Notice that all the media response curves are S shaped which assumes that there are threshold effects associated with each of these media variables. For the moment we assume for simplicity that the cost per impression is the same for each of these media variables. This facilitates the discussion since it implies that maximizing profits is equivalent to maximizing total sales from a combination of these media variables. We will also assume that each media variable is associated with carry over effects so that each media not only affects sales in the week in which it is aired but also in several subsequent weeks.
The problem then resorts to finding the optimal mix of these variables that maximize total sales.

This problem, as we have defined it, has several local optima and one global optimum as shown by the surface chart in figure 2. The 2 horizontal axes of the chart shows the impressions of x and y and the vertical axis displays the total sales. We assume as before that the cost per impression is the same for each of these media variables and equals 1 unit per impression. The total budget is assumed to be fixed at 205 units so that x + y + z <= 205 must always hold. The global optimum involves using 48 units of x, 94 units of y and 63 units of z. The corresponding maximum sales is $10.7 million. As mentioned before this problem also has several local optima. One local optima involves investing in 124 units of y and 81 units of z with sales of $8.3 million. Another local optima involves spending 68 units on x and 137 units on y resulting in a total sales of $7.9 million. The corner solution where we invest 205 units on y and nothing on x and z is also a local optima since marginal changes from this point does not lead to increases in sales. Depending on the initial conditions that we input in the problem, we may get any one of the several local optima or the global one as our final solution. With a large number of media response variables or more non-linearities in the response function, the number of local optima will multiply increasing our chances of obtaining a local optimum as the final solution instead of the global one.

Figure 1

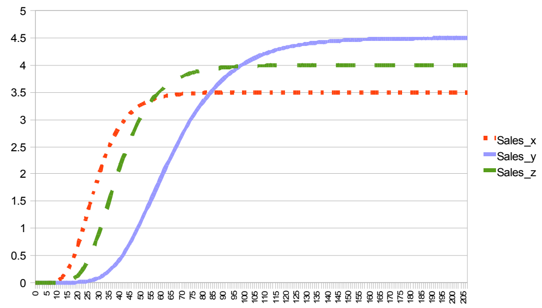
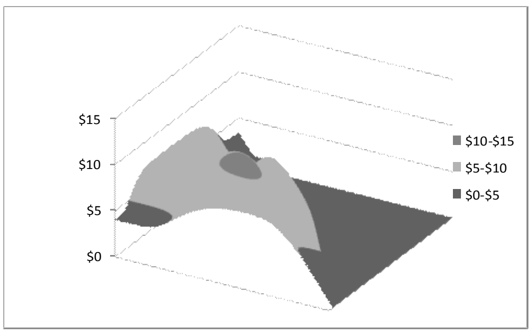


Figure 2



In the following section we show how, using our algorithm we obtain a file that displays the total sales for each mix of media variables. The global optimum can then be easily read of from the list. Besides, total sales for other combinations of the media variables can also be easily obtained. This may be useful if during the planning stage, the media planners need to look at various scenarios and choose a feasible option based on different constraints.

4. A MORE UNIVERSAL METHOD:

We propose an alternative solution, using a SAS macro, which can be used to obtain a close to global solution for the optimal media mix. The macro uses an iteration method to obtain the value of the objective function for all potential allocations. The allocation that maximizes the objective function can then be chosen as the optimal media mix. Since the macro works with the transformed variables, non linear transformation and all sorts of complicated carryover effects can be incorporated within the macro. Furthermore, since the algorithm does not require solving for first derivatives, the objective function is not required to be differentiable. Below, we outline the steps of the macro and provide a description of how each step works.

We assume that the Sales equation for the product in question is given by equation (1). Equation (1) says that sales per week is determined by the amount of advertisement exposures through television, magazine and online channels. In reality we would expect several other terms to enter the above equation, such as influence of seasonality, holidays, macroeconomic factors, other marketing variables, competitor effects etc. However for simplicity we have left those terms out of the equation and they are assumed to remain at their original values irrespective of the mix between the media variables. Therefore when we refer to Sales, the reader should keep in mind that it really means incremental sales arising from media advertisement over and above baseline sales that would result if the company made no media investments whatsoever. To keep things simple, we have assumed that there are no interaction effects between the media variables and any other variables in the model. In reality such interaction effects exist and are modeled and only add to the non-linearities and non-convexities of the model further providing support for this analysis. Note that while the media variables in the model are assumed to be impressions, the analysis is similar if media spending is used in the model instead of media impressions. The process by which we arrive at these variable transformations and the final model in equation (1) – (3) is beyond the scope of this paper and for our purpose we simply assume that the final model has already been obtained and tested and has been found to be satisfactory. We will use this model to explain the procedure of obtaining the optimal marketing mix. For simplicity we assume that we are trying to allocate our resources between only three media variables (Magazine, Television and Online). We also assume that there are threshold effects associated with each media variable so that the relationship between each media variable and sales is S-shaped i.e. has an initial convex and subsequently a concave section. Several S shaped functions can be used to depict this relationship. We will choose a specific function called the Gompertz function which is a non-linear S shaped function and will be beneficial for demonstration purposes. Assume that our data consists of weekly TV GRPs and Magazine impressions and Online impressions as well as product sales for about 3-4 years. Suppose that the final model can be represented by the following equation :

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Sales = A*b^(a^f(TV)) +
C*d^(c^g(Mag)) + E*f^(e^h(Onl))
                    ............................................... (1)
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where f(TV) and g(Mag) and h(Onl) are transformations of the TV, Print and Online variables respectively that incorporate carry over effects.

The carryover effects are assumed to be as follows. For each media variable we assume that only a certain fraction of the consumers who see an ad in a particular week actually go to the store to purchase the product in that week. The other consumers will visit the store in one of the following weeks. We assume that 60% of the consumers who see a TV ad will visit the store during the week the advertisement is aired. Therefore the transformed TV variable taking into account carryover effects can be obtained as follows:

f(TV) = TV60 = 0.6*TV + 0.4*lag(TV60).

Similarly, the transformed magazine and Online variables are as follows :

g(Mag) = Mag40 = 0.4*Mag + 0.6*lag(Mag40).

h(Onl) = Onl30 = 0.3*Mag + 0.7*lag(Onl30).

Therefore in explicit terms equation (1) can now be written as :

Sales = A*b^(a^TV60) + C*d^(c^Mag50) + E*f^(e^Onl30) ………………………………… (2)

We assume that the above non-linear regression model has been solved and begets the following coefficients for a, b, c, d, e,f, A. C and E : a=0.9, b=0.000001, c = 0.95 , d=0.000000009 , e= 0.92, f= 0.00000001 , A=3.5 , C=4.5 , E= 4 .

Substituting these parameter values in equation(2) gives the following model to estimate product Sales in any week:

Sales = 3.5*0.000001^(0.9^TV60) + 4.5*0.000000009^(0.95^Mag50) + 4*0.00000001^(0.92^Onl30) ……… (3)

4.1. HOW THE ALGORITHM WORKS:

Assume that during the last year, approximately 60% of the budget was allocated to TV and 20% to Magazine and 20% to Online. Assuming the cost per point and flighting remains the same as last year, we can try to see the effect on sales if the resources were reallocated to devote say k% to TV, l% to Magazine and (100 – k - l)% to Online. For every value of k and l, we have a separate budget for each media variable. If this budget is less (or greater) than the original budget, the algorithm prorates the media variables to a fraction (or a multiple) of its original value. The new values of the media variables are the amounts that could have been purchased at the new budget for each. We assume that the flighting remains exactly the same as before so that the entire media stream is prorated in exactly the same manner in each week. For our purposes let us assume that for k = k' and l = l' , the new prorated TV, Magazine and Online variables are TV', Magazine' and Online' respectively. Therefore, as an example if k' = 40% then TV' = (TV*40)/60. Once these new TV, Magazine and Online variables have been obtained, the algorithm next transforms these new variables to obtain TV60, Mag50 and Onl30 respectively. The sales resulting from this allocation of the budget between TV, Magazine and Online is then easily calculated from equation (1).

The algorithm repeats all of the above steps for all values of k and l (= 1, 2, ….100) and obtains the corresponding estimated Sales. The estimated Sales figures for different budget allocations can then be compared and the best one which makes the most business sense can be chosen.

While the example above explains the algorithm with only three media variables, the macro can accommodate additional variables. Naturally, as the number of variables increase, the algorithm works through a larger number of iterations which increases the time needed for the macro to run.

5. CONCLUSION

The paper looks at the problem of optimizing the marketing mix subject to resource constraints. Non linear demand curves and lagged (or lead) effects of advertising along with interaction effects often lead to non convex objective functions in the marketing mix modeling world. Standard optimization methods need the objective functions to be convex and differentiable and cannot be applied when these complexities exist in the model. The algorithm outlined in this paper can help to find the global optimum in these situations when standard methods cannot be applied. Besides, this algorithm helps the marketing planner to look at various scenarios and choose the one that makes the most business sense.

6. REFERENCES

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